1.1

1) $AllZero = \{ [\langle f_0, s_0\rangle, \dots, \langle f_n, s_n\rangle] \mid \forall 0 \le i < n.\ \langle f_i, s_i\rangle \to \langle f_{i+1}, s_{i+1}\rangle \wedge s_n = skip \wedge \forall x \in dom(f_n).\ f_n(x) = 0 \}$

2) $MonX = \{ [\langle f_0, s_0\rangle, \dots, \langle f_n, s_n\rangle] \mid \forall 0 \le i < n.\ \langle f_i, s_i\rangle \to \langle f_{i+1}, s_{i+1}\rangle \wedge s_n = skip \wedge (\forall_{0 \le i < n}.\ x \in dom(f_i) \cap dom(f_{i+1}) \Rightarrow f_{i+1}(x) \ge f_i(x)) \}$

3) $AllMon = \{ [\langle f_0, s_0\rangle, \dots, \langle f_n, s_n\rangle] \mid \forall 0 \le i < n.\ \langle f_i, s_i\rangle \to \langle f_{i+1}, s_{i+1}\rangle \wedge s_n = skip \wedge (\forall_{0 \le i < n}.\ \forall x \in dom(f_i) \cap dom(f_{i+1}).\ f_{i+1}(x) \ge f_i(x)) \}$

4) $XGreaterThanY = \{ [\langle f_0, s_0\rangle, \dots, \langle f_n, s_n\rangle] \mid \forall 0 \le i < n.\ \langle f_i, s_i\rangle \to \langle f_{i+1}, s_{i+1}\rangle \wedge s_n = skip \wedge (\forall_{0 \le i \le n}.\ \{x, y\} \subseteq dom(f_i) \Rightarrow f_i(x) > f_i(y)) \}$

5) $XGreaterThanAll = \{ [\langle f_0, s_0\rangle, \dots, \langle f_n, s_n\rangle] \mid \forall 0 \le i < n.\ \langle f_i, s_i\rangle \to \langle f_{i+1}, s_{i+1}\rangle \wedge s_n = skip \wedge (\forall_{0 \le i \le n}.\ x \in dom(f_i) \Rightarrow \forall y \in dom(f_i) \setminus \{x\}.\ f_i(x) > f_i(y)) \}$

6) $YBookKeepX = \{ [\langle f_0, s_0\rangle, \dots, \langle f_n, s_n\rangle] \mid \forall 0 \le i < n.\ \langle f_i, s_i\rangle \to \langle f_{i+1}, s_{i+1}\rangle \wedge s_n = skip$

$\wedge \neg(\exists i, j.\ i \ne j \wedge sgn(f_i(x)) \ne sgn(f_{i+1}(x)) \wedge sgn(f_j(x)) \ne sgn(f_{j+1}(x)))$

$\wedge (\forall 0 \le i < n.\ sgn(f_i(x)) \ne sgn(f_{i+1}(x)) \Rightarrow \exists i \le j \le n.\ \forall j \le k \le n.\ f_k(y) = f_i(x)) \}$

- 1-5 safety properties
- 6 liveness property

1.2

1) AllZero: skip

!AllZero: $x := 1$

2) 3) MonX, AllMon: $x := 0;\ x := 1$

!MonX, !AllMon: $x := 1;\ x := 0$

4) 5) XGreaterThanY, XGreaterThanAll: $x := 3;\ y := 2$

!XGreaterThanY, !XGreaterThanAll: $x := 2;\ y := 3$

6) YBookKeepX: $x := 1;\ x := -1;\ y := 1$

!YBookKeepX: $x := 1;\ y := -1;\ x := -1$

2.1)

1) $s \equiv if(h)\ \{\ l := l+z\ \}\ else\ \{\ skip\ \}$

$\mathcal{C}(s) = assume(l_1 = l_2 \wedge z_1 = z_2);$
$\quad if(h_1)\ \{\ l_1 := l_1 + z_1\ \}\ else\ \{\ skip\ \};$
$\quad if(h_2)\ \{\ l_2 := l_2 + z_2\ \}\ else\ \{\ skip\ \};$
$\quad assert(l_1 = l_2 \wedge z_1 = z_2)$

$True,\ [l_1 \mapsto \hat{l}_1, l_2 \mapsto \hat{l}_2, h_1 \mapsto \hat{h}_1, h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\big\downarrow\ assume(l_1 = l_2 \wedge z_1 = z_2)$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2,\ [l_1 \mapsto \hat{l}_1, l_2 \mapsto \hat{l}_2, h_1 \mapsto \hat{h}_1, h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\xrightarrow{\hat{h}_1 \neq 0}$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 \neq 0,\ [l_1 \mapsto \hat{l}_1, l_2 \mapsto \hat{l}_2, h_1 \mapsto \hat{h}_1, h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\big\downarrow\ l_1 = l_1 + z_1$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 \neq 0,$
$[l_1 \mapsto \hat{l}_1 + \hat{z}_1, l_2 \mapsto \hat{l}_2,$
$h_1 \mapsto \hat{h}_1, h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\xrightarrow{\hat{h}_2 \neq 0}$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 \neq 0 \wedge \hat{h}_2 \neq 0,$
$[l_1 \mapsto \hat{l}_1 + \hat{z}_1, l_2 \mapsto \hat{l}_2,$
$h_1 \mapsto \hat{h}_1, h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\big\downarrow\ l_2 := l_2 + z_2$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 \neq 0 \wedge \hat{h}_2 \neq 0,$
$[l_1 \mapsto \hat{l}_1 + \hat{z}_1, l_2 \mapsto \hat{l}_2 + \hat{z}_2,$
$h_1 \mapsto \hat{h}_1, h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\big\downarrow\ assert(l_1 = l_2 \wedge z_1 = z_2)$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 \neq 0 \wedge \hat{h}_2 \neq 0$
$\Rightarrow \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 + \hat{z}_1 = \hat{l}_2 + \hat{z}_2$
$\leadsto$
$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 \neq 0 \wedge \hat{h}_2 \neq 0$
$\wedge ((\hat{z}_1 \neq \hat{z}_2) \vee (\hat{l}_1 + \hat{z}_1 \neq \hat{l}_2 + \hat{z}_2))$

UNSAT

$\xrightarrow{\hat{h}_2 = 0}$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 \neq 0 \wedge \hat{h}_2 = 0,$
$[l_1 \mapsto \hat{l}_1 + \hat{z}_1, l_2 \mapsto \hat{l}_2,$
$h_1 \mapsto \hat{h}_1, h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\big\downarrow\ skip$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 \neq 0 \wedge \hat{h}_2 = 0,$
$[l_1 \mapsto \hat{l}_1 + \hat{z}_1, l_2 \mapsto \hat{l}_2,$
$h_1 \mapsto \hat{h}_1, h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\big\downarrow\ assert(l_1 = l_2 \wedge z_1 = z_2)$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 \neq 0 \wedge \hat{h}_2 = 0$
$\Rightarrow \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 + \hat{z}_1 = \hat{l}_2$
$\leadsto$
$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 \neq 0 \wedge \hat{h}_2 = 0$
$\wedge ((\hat{z}_1 \neq \hat{z}_2) \vee (\hat{l}_1 + \hat{z}_1 \neq \hat{l}_2))$

$\boxed{\hat{l}_1, \hat{l}_2 \mapsto 1 \quad \hat{h}_1 \mapsto 1 \quad \hat{h}_2 \mapsto 0 \quad \hat{z}_1, \hat{z}_2 \mapsto 1}$ SAT

$\xrightarrow{\hat{h}_1 = 0}$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 = 0,\ [l_1 \mapsto \hat{l}_1, l_2 \mapsto \hat{l}_2, h_1 \mapsto \hat{h}_1, h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\big\downarrow\ skip$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 = 0,\ [l_1 \mapsto \hat{l}_1, l_2 \mapsto \hat{l}_2, h_1 \mapsto \hat{h}_1, h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\xrightarrow{\hat{h}_2 \neq 0}$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 \neq 0,$
$[l_1 \mapsto \hat{l}_1, l_2 \mapsto \hat{l}_2, h_1 \mapsto \hat{h}_1,$
$h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\big\downarrow\ l_2 = l_2 + z_2$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 \neq 0,$
$[l_1 \mapsto \hat{l}_1, l_2 \mapsto \hat{l}_2 + \hat{z}_2, h_1 \mapsto \hat{h}_1,$
$h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\big\downarrow\ assert(l_1 = l_2 \wedge z_1 = z_2)$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 \neq 0$
$\Rightarrow (\hat{z}_1 = \hat{z}_2 \ \wedge\ \hat{l}_1 = \hat{l}_2 + \hat{z}_2)$
$\leadsto$
$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 \neq 0$
$\wedge\ (\hat{z}_1 \neq \hat{z}_2 \vee \hat{l}_1 \neq \hat{l}_2 + \hat{z}_2)$

$\boxed{\hat{l}_1, \hat{l}_2 \mapsto 1 \quad \hat{h}_1 \mapsto 1 \quad \hat{h}_2 \mapsto 0 \quad \hat{z}_1, \hat{z}_2 \mapsto 1}$ SAT

$\xrightarrow{\hat{h}_2 = 0}$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 = 0$
$[l_1 \mapsto \hat{l}_1, l_2 \mapsto \hat{l}_2, h_1 \mapsto \hat{h}_1,$
$h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\big\downarrow\ skip$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 = 0$
$[l_1 \mapsto \hat{l}_1, l_2 \mapsto \hat{l}_2, h_1 \mapsto \hat{h}_1,$
$h_2 \mapsto \hat{h}_2, z_1 \mapsto \hat{z}_1, z_2 \mapsto \hat{z}_2]$

$\big\downarrow\ assert(l_1 = l_2 \wedge z_1 = z_2)$

$\hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 = 0$
$\Rightarrow \hat{l}_1 = \hat{l}_2 \wedge \hat{z}_1 = \hat{z}_2$ ✓

22 -

if (h) { x := x+z } else {skip}
if (!h) { y := y+z } else {skip}
x:=0; y:=0; l := x+y

$\rightsquigarrow$

assume$(x_1 = x_2 \land z_1 = z_2 \land l_1 = l_2 \land y_1 = y_2)$;
if (h_1) { $x_1 := x_1+z_1$ } else {skip};
if $(!h_1)$ { $y_1 := y_1+z_1$ } else {skip};
$x_1 := 0$; $y_1 := 0$; $l_1 := x_1+y_1$;
if (h_2) { $x_2 := x_2+z_2$ } else {skip};
if $(!h_2)$ { $y_2 := y_2+z_2$ } else {skip};
$x_2 := 0$; $y_2 := 0$; $l_2 := x_2+y_2$;
assert$(x_1 = x_2 \land z_1 = z_2 \land l_1 = l_2 \land y_1 = y_2)$

$\text{true}, [x_1 \mapsto \hat{x}_1, y_1 \mapsto \hat{y}_1, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{l}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ assume$(x_1 = x_2 \land z_1 = z_2 \land l_1 = l_2 \land y_1 = y_2)$

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2, [x_1 \mapsto \hat{x}_1, y_1 \mapsto \hat{y}_1, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{l}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

Branch $\hat{h}_1 = 0$:

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 = 0, [x_1 \mapsto \hat{x}_1, y_1 \mapsto \hat{y}_1, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{l}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ skip

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 = 0, [x_1 \mapsto \hat{x}_1, y_1 \mapsto \hat{y}_1, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{l}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ $y_1 := y_1 + z_1$

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 = 0, [x_1 \mapsto \hat{x}_1, y_1 \mapsto \hat{y}_1+\hat{z}_1, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{l}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ $x_1 := 0$; $y_1 := 0$; $l_1 := x_1+y_1$;

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 = 0, [x_1 \mapsto 0, y_1 \mapsto 0. z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1+\hat{y}_1+\hat{z}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

Branch $\hat{h}_2 = 0$:

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 = 0 \land \hat{h}_2 = 0, [x_1 \mapsto 0, y_1 \mapsto 0. z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1+\hat{y}_1+\hat{z}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ skip

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 = 0 \land \hat{h}_2 = 0, [x_1 \mapsto 0, y_1 \mapsto 0. z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1+\hat{y}_1+\hat{z}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

Branch $\hat{h}_2 \neq 0$:

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 = 0 \land \hat{h}_2 \neq 0, [x_1 \mapsto 0, y_1 \mapsto 0. z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1+\hat{y}_1+\hat{z}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ $x_2 := x_2 + z_2$

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 = 0 \land \hat{h}_2 \neq 0, [x_1 \mapsto 0, y_1 \mapsto 0. z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1+\hat{y}_1+\hat{z}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2+\hat{z}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

Branch $\hat{h}_1 \neq 0$:

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 \neq 0, [x_1 \mapsto \hat{x}_1, y_1 \mapsto \hat{y}_1, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{l}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ $x_1 := x_1 + z_1$

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 \neq 0, [x_1 \mapsto \hat{x}_1+\hat{z}_1, y_1 \mapsto \hat{y}_1, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{l}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ skip

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 \neq 0, [x_1 \mapsto \hat{x}_1+\hat{z}_1, y_1 \mapsto \hat{y}_1, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{l}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ $x_1 := 0$; $y_1 := 0$; $l_1 := x_1+y_1$;

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 \neq 0, [x_1 \mapsto 0, y_1 \mapsto 0. z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1+\hat{y}_1+\hat{z}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

Branch $\hat{h}_2 = 0$:

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 \neq 0 \land \hat{h}_2 = 0, [x_1 \mapsto 0, y_1 \mapsto 0. z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1+\hat{y}_1+\hat{z}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ skip

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_1 \neq 0 \land \hat{h}_2 = 0, [x_1 \mapsto 0, y_1 \mapsto 0. z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1+\hat{y}_1+\hat{z}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

Branch $\hat{h}_2 \neq 0$:

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_2 \neq 0 \land \hat{h}_1 \neq 0, [x_1 \mapsto 0, y_1 \mapsto 0. z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1+\hat{y}_1+\hat{z}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ $x_2 := x_2 + z_2$

$\hat{x}_1 = \hat{x}_2 \land \hat{y}_1 = \hat{y}_2 \land \hat{z}_1 = \hat{z}_2 \land \hat{l}_1 = \hat{l}_2 \land \hat{h}_2 \neq 0 \land \hat{h}_1 \neq 0, [x_1 \mapsto 0, y_1 \mapsto 0. z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1+\hat{y}_1+\hat{z}_1, h_1 \mapsto \hat{h}_1, x_2 \mapsto \hat{x}_2+\hat{z}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ skip

$\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 = \hat{l}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 = 0$
$[x_1 \mapsto 0, y_1 \mapsto 0, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1 + \hat{y}_1 + \hat{z}_1, h_1 \mapsto \hat{h}_1,$
$x_2 \mapsto \hat{z}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow x_2 := x_2 + z_2$

$\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 = \hat{l}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 \neq 0$
$[x_1 \mapsto 0, y_1 \mapsto 0, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1 + \hat{y}_1 + \hat{z}_1, h_1 \mapsto \hat{h}_1,$
$x_2 \mapsto \hat{z}_2 + \hat{z}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ skip

$\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 = \hat{l}_2 \wedge \hat{h}_1 \neq 0 \wedge \hat{h}_2 = 0,$
$[x_1 \mapsto 0, y_1 \mapsto 0, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1 + \hat{y}_1 + \hat{z}_1, h_1 \mapsto \hat{h}_1,$
$x_2 \mapsto \hat{z}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow x_2 := x_2 + z_2$

$\hat{h}_2 \neq 0 \wedge$

$\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 = \hat{l}_2 \wedge \hat{h}_1 \neq 0,$
$[x_1 \mapsto 0, y_1 \mapsto 0, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1 + \hat{y}_1 + \hat{z}_1, h_1 \mapsto \hat{h}_1,$
$x_2 \mapsto \hat{z}_2 + \hat{z}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow y_2 := y_2 + z_2$

$\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 = \hat{l}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 = 0$
$[x_1 \mapsto 0, y_1 \mapsto 0, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1 + \hat{y}_1 + \hat{z}_1, h_1 \mapsto \hat{h}_1,$
$x_2 \mapsto \hat{z}_2, y_2 \mapsto \hat{y}_2 + \hat{z}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ skip

$\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 = \hat{l}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 \neq 0$
$[x_1 \mapsto 0, y_1 \mapsto 0, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1 + \hat{y}_1 + \hat{z}_1, h_1 \mapsto \hat{h}_1,$
$x_2 \mapsto \hat{z}_2 + \hat{z}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow y_2 := y_2 + z_2$

$\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 = \hat{l}_2 \wedge \hat{h}_1 \neq 0 \wedge \hat{h}_2 = 0,$
$[x_1 \mapsto 0, y_1 \mapsto 0, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1 + \hat{y}_1 + \hat{z}_1, h_1 \mapsto \hat{h}_1,$
$x_2 \mapsto \hat{z}_2, y_2 \mapsto \hat{y}_2 + \hat{z}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow$ skip

$\hat{h}_2 \neq 0 \wedge$

$\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 = \hat{l}_2 \wedge \hat{h}_1 \neq 0,$
$[x_1 \mapsto 0, y_1 \mapsto 0, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1 + \hat{y}_1 + \hat{z}_1, h_1 \mapsto \hat{h}_1,$
$x_2 \mapsto \hat{z}_2 + \hat{z}_2, y_2 \mapsto \hat{y}_2, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{l}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow x_2 := 0;\ y_2 := 0;\ l_2 := x_2 + y_2$

$\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 = \hat{l}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 = 0$
$[x_1 \mapsto 0, y_1 \mapsto 0, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1 + \hat{y}_1 + \hat{z}_1, h_1 \mapsto \hat{h}_1,$
$x_2 \mapsto 0, y_2 \mapsto 0, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{x}_2 + \hat{y}_2 + \hat{z}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow x_2 := 0;\ y_2 := 0;\ l_2 := x_2 + y_2$

$\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 = \hat{l}_2 \wedge \hat{h}_1 = 0 \wedge \hat{h}_2 \neq 0$
$[x_1 \mapsto 0, y_1 \mapsto 0, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1 + \hat{y}_1 + \hat{z}_1, h_1 \mapsto \hat{h}_1,$
$x_2 \mapsto 0, y_2 \mapsto 0, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{x}_2 + \hat{y}_2 + \hat{z}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow x_2 := 0;\ y_2 := 0;\ l_2 := x_2 + y_2$

$\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 = \hat{l}_2 \wedge \hat{h}_1 \neq 0 \wedge \hat{h}_2 = 0$
$[x_1 \mapsto 0, y_1 \mapsto 0, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1 + \hat{y}_1 + \hat{z}_1, h_1 \mapsto \hat{h}_1,$
$x_2 \mapsto 0, y_2 \mapsto 0, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{x}_2 + \hat{y}_2 + \hat{z}_2, h_2 \mapsto \hat{h}_2]$

$\downarrow x_2 := 0;\ y_2 := 0;\ l_2 := x_2 + y_2$

$\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_2 \wedge \hat{z}_1 = \hat{z}_2 \wedge \hat{l}_1 = \hat{l}_2 \wedge \hat{h}_1 \neq 0 \wedge \hat{h}_2 \neq 0$
$[x_1 \mapsto 0, y_1 \mapsto 0, z_1 \mapsto \hat{z}_1, l_1 \mapsto \hat{x}_1 + \hat{y}_1 + \hat{z}_1, h_1 \mapsto \hat{h}_1,$
$x_2 \mapsto 0, y_2 \mapsto 0, z_2 \mapsto \hat{z}_2, l_2 \mapsto \hat{x}_2 + \hat{y}_2 + \hat{z}_2, h_2 \mapsto \hat{h}_2]$

$$\boxed{\hat{x}_1 = \hat{x}_2 \wedge \hat{y}_1 = \hat{y}_0 \wedge \hat{z}_1 = \hat{z}_2 \Rightarrow \hat{x}_1 + \hat{y} + \hat{z}_1 = \hat{x}_2 + \hat{y}_2 + \hat{z}_2}$$

holds

3.1) $s \in NI(\Gamma) \Rightarrow [\![C(s)]\!] \subseteq T(\Gamma)$

Assume $s \in NI(\Gamma)$ (H1)

Suppose $[\![C(s)]\!] \not\subseteq T(\Gamma)$ (H2)

From H2, we conclude that there is a finite trace of $C(s)$ that is not in $T(\Gamma)$.

Let $[\langle p_0, s_0 \rangle, \ldots, \langle p_n, s_n \rangle]$ be that trace